Source task and conventions

Compute $CC^\top$ and interpret its diagonal.

Conventional solution and proof

For $C = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$, row inner products give

$$CC^\top = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

For a binary incidence matrix, $(CC^\top)_{ii} = \sum_j C_{ij}^2 = \sum_j C_{ij}$ is the number of documents containing term i, its document frequency. Off-diagonal entry (i, l) counts documents containing both terms. The diagonal interpretation uses binarity; for term-frequency counts it becomes a sum of squared frequencies.

Structural or analytic derivation

The co-occurrence matrix is a Gram matrix of term-incidence vectors, so it is PSD and its diagonal counts arise directly as squared feature norms.

Comparative cost and preferred method

If only matrix-vector actions are needed, use $C(C^\top v)$ without storing the full co-occurrence matrix. With sparse incidence this costs $O(\operatorname{nnz} C)$ per action. The question asks all nine entries, for which the direct product is the appropriate small computation.