Source task and conventions

Verify every SVD property using exact radicals in place of rounded decimals.

Conventional solution and proof

The exact matrices corresponding to the printed values are

$$U = \begin{pmatrix} -2/\sqrt{6} & 0 \\ -1/\sqrt{6} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{2} \end{pmatrix}, \quad \Sigma = \operatorname{diag}(\sqrt{3}, 1), \quad V^\top = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}.$$

The squared norms of the columns of U are $(4+1+1)/6 = 1$ and $(0+1+1)/2 = 1$, and their inner product is zero. The rows of $V^\top$ likewise have unit norm and zero mutual inner product. The singular values are nonnegative and decreasing. Multiplying the two rank-one terms gives

$$\sqrt{3} u_1 v_1^\top = \begin{pmatrix} 1 & 1 \\ 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}, \quad u_2 v_2^\top = \begin{pmatrix} 0 & 0 \\ -1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix},$$

whose sum is C from Equation (18.12). Thus $C = U\Sigma V^\top$ and $U^\top U = V^\top V = I_2$. This is a thin SVD; $UU^\top$ is a rank-two projector, not I_3. For a full SVD append $u_3 = (-1, 1, 1)^\top/\sqrt{3}$ and a zero row to Σ. The rounded decimals in the source satisfy these equalities only approximately.

Structural or analytic derivation

The exact verification supplies all orthogonality and reconstruction certificates and resolves the distinction between full and reduced SVD conventions.