Source task and conventions

Write both singular-value matrices for the 4×2 example.

Conventional solution and proof

Example 18.3 has $C = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 1 \end{pmatrix}$. Its column Gram matrix is $C^\top C = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$, with eigenvalues $5, 1$, so its singular values are $\sqrt{5}, 1$. In a full SVD with $U \in \mathbb{R}^{4\times 4}$ and $V \in \mathbb{R}^{2\times 2}$,

$$\Sigma_2 = \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \Sigma_2' = \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 1 \end{pmatrix}.$$

The reduced representation uses the first two columns of U, so it yields exactly the same product. The example already displays a reduced SVD; its printed 2×2 array is therefore Σ_2'. Distinguishing that convention explains why no additional zero rows appear in the example itself.

Structural or analytic derivation

For a general rank-k truncation, $C_k = \sum_{j=1}^{k} \sigma_j u_j v_j^\top = U_k' \Sigma_k' V_k'^\top$, because every omitted diagonal entry in the full rectangular array is zero. Entrywise expansion proves the equality.

Comparative cost and preferred method