Structural or analytic derivation

The factorization $\widetilde{K} = (PX)^\top (PX)$ proves symmetry, positivity, the residual sign and exactness in one representation. A thin factor $B = CV_r\Sigma_r^{-1}$ gives $\widetilde{K} = BB^\top$ and can replace $W^\dagger$ in subsequent operations. The identity remains valid for singular W; an ordinary inverse would fail in that case.

Comparative cost and preferred method

The full matrix has m^2 entries; the stated approximation representation has $ml + l^2$. Their ratio is $m^2/(ml + l^2) \approx 1999.0005$. Matrix-vector products can use $B(B^\top v)$ in $O(mr)$ arithmetic after factor construction, compared with $O(m^2)$ for a stored dense matrix. This is an exact representation of the approximation, not a claim that arbitrary landmarks have small error. When the rank condition in (d) holds, the representation is exact. Landmark choice, kernel evaluation, factorization and external-memory costs must be included in an application.