Source task and conventions

Prove positive semidefiniteness of $K(x,y)/(\alpha(x)\alpha(y))$ for strictly positive α.

Conventional solution and proof

For arbitrary points $x_1,\dots,x_m$ and real coefficients c_i, put $d_i = c_i/\alpha(x_i)$. Symmetry is inherited from K, and

$$\sum c_i c_j K'(x_i,x_j) = \sum d_i d_j K(x_i,x_j) \geq 0.$$