The denominators exist because $\alpha(x) > 0$. This verifies the defining quadratic-form condition for every finite sample, including repeated points.

## Structural or analytic derivation

If $K(x, y) = \langle \phi(x), \phi(y) \rangle$, use $\phi'(x) = \phi(x)/\alpha(x)$. Equivalently the Gram matrix is the congruence $D^{-1}GD^{-1}$, where $D = \mathrm{diag}(\alpha(x_i))$. A congruence preserves positive semidefiniteness.

## Comparative cost and preferred method

The symbolic certificate is independent of sample size. Scaling an already stored $m \times m$ Gram matrix costs $O(m^2)$ arithmetic; testing it by a generic eigensolver costs $O(m^3)$ and only checks that one matrix. Both proofs are elementary and equally short; no general computational speedup in evaluating the kernel is implied.