(g) For $a > 0$ and $s \geq 0$,

$$e^{-a\sqrt{s}} = \frac{a}{2\sqrt{\pi}} \int_0^\infty t^{-3/2} e^{-a^2/(4t)} e^{-ts}\, dt.$$

Here is a proof of the integral identity. Substituting $u = a/(2\sqrt{t})$ reduces the right side to $2I(b)/\sqrt{\pi}$, where $b = a\sqrt{s}/2$ and $I(b) = \int_0^\infty e^{-u^2 - b^2/u^2}\, du$. For $b > 0$, differentiation under the integral and the substitution $v = b/u$ give $I'(b) = -2I(b)$. Moreover $I(0) = \sqrt{\pi}/2$ and dominated convergence gives continuity at zero. Thus $I(b) = (\sqrt{\pi}/2)e^{-2b}$. With $a = 1/\sigma$, $s = \|x - y\|^2$, the positive Gaussian mixture proves (g).

(h) Use $\phi_x(t) = \mathbf{1}_{[0,x]}(t) - x$ in $L^2[0,1]$. Expanding its inner product gives $\min(x,y) - xy$; endpoint choices on null sets are immaterial.