Source task and conventions

Determine whether $|x - y|$ is PDS.

Conventional solution and proof

Choose points $0, 1$. The Gram matrix is $G = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. For $c = (1, -1)^\top$, $c^\top G c = -2 < 0$, so the kernel is not PDS. Equivalently, its eigenvalues are $1, -1$. A general PSD matrix with a zero diagonal entry must have the corresponding row zero, because its 2×2 principal minors satisfy $|G_{ij}|^2 \leq G_{ii} G_{jj}$; this kernel violates that necessary condition.

Structural or analytic derivation

The counterexample is already minimal: one nonnegative diagonal value cannot disprove PDS, while two distinct points do. This obstruction is more economical than constructing a large sampled Gram matrix.

Comparative cost and preferred method

Two points and one quadratic form certify failure for the entire proposed kernel. The conventional counterexample is the preferred method; introducing contour integrals would add no information needed by the question.