Source task and conventions

Prove the source claim for $K(x, y) = 1 - \langle x, y \rangle$.

Conventional solution and proof

For real coefficients with $\sum_i c_i = 0$,

$$\sum_{i,j} c_i c_j [1 - \langle x_i, x_j \rangle] = (\sum_i c_i)^2 - \left\| \sum_i c_i x_i \right\|^2 \leq 0.$$

Symmetry holds in a real Hilbert space. In a complex Hilbert space, use Hermitian symmetry and coefficients $c_i \in \mathbb{C}$ with $\sum_i c_i = 0$, replacing the quadratic form by $\sum_{i,j} \overline{c}_i c_j K(x_i, x_j)$ in a consistent inner-product convention. The same norm identity applies.

Structural or analytic derivation

The constant kernel is annihilated on the zero-sum subspace, leaving the negative Gram form. No finite-dimensional approximation to the Hilbert space is required.

Comparative cost and preferred method

The certificate uses one vector sum and one squared norm. For m vectors of finite dimension d, this is $O(md)$ instead of explicitly forming m^2 pairwise inner products at $O(m^2 d)$. Ordinary unrestricted negative semidefiniteness would be a different and generally false assertion.