Structural or analytic derivation

The fitted response is the unique orthogonal projection of y onto $\operatorname{ran} X^\top$. The coefficient ambiguity is a whole affine nullspace. This distinction prevents a singular inverse from being treated as an admissible formula.

Comparative cost and preferred method

A rank-revealing SVD or QR supplies the solution and its complete nullspace certificate in one factorization. Normal-equation inversion is inadmissible when rank deficient and squares the condition number when full rank. A suitable conventional rank-revealing factorization is already the preferred algorithm; the gain is a complete characterization, not a new asymptotic improvement.