Source task and conventions

Prove the dual-parameter perturbation bound and explicitly adjudicate the printed sign.

Conventional solution and proof

Put $A = K + \lambda I$, $A' = K' + \lambda I$, with $K, K' \succeq 0$, $\lambda > 0$, and $\alpha = A^{-1}y$, $\alpha' = A'^{-1}y$. Multiplying out gives

$$A'^{-1} - A^{-1} = A'^{-1}(A - A')A^{-1},$$

hence

$$\boxed{\alpha' - \alpha = -A'^{-1}(K' - K)A^{-1}y.}$$

This has the opposite sign from the displayed identity in part (a); the source hint has the correct minus sign. A scalar counterexample is $K = 0, K' = 1, \lambda = y = 1$: the actual difference is $-1/2$, while the printed right side is $+1/2$. For (b), spectral positivity gives $\|A^{-1}\|_2, \|A'^{-1}\|_2 \le 1/\lambda$. If $|y_i| \le M$, then $\|y\|_2 \le \sqrt{m}M$. Submultiplicativity therefore yields

$$\|\alpha' - \alpha\|_2 \le \lambda^{-2}\|K' - K\|_2\sqrt{m}M.$$

These arguments include singular unregularized K, since adding λI makes both matrices positive definite.

Structural or analytic derivation

The identity is an exact first-resolvent identity, not a first-order approximation. It also gives the sharper a posteriori bound $\|\alpha' - \alpha\| \le \|K' - K\|\,\|\alpha\|/\lambda$, and bounds using actual smallest eigenvalues can replace λ^{-2}. If $\Delta K = UV^\top$ has small rank, the Woodbury update follows by multiplying the proposed inverse and checking the identity, provided $I + V^\top A^{-1}U$ is invertible.