Source task and conventions

Complete all four parts with a fixed unnormalised regularization convention and the corrected denominator.

Conventional solution and proof

Use the RKHS objective $J_T(h) = \sum_{(x,y)\in T}(h(x)-y)^2 + \lambda\|h\|^2$, $\lambda > 0$. It is strictly convex and coercive, with a unique minimizer. For (a), replace the ith label by $h_{S_i}(x_i)$ (the printed $h_{S_i}(y_i)$ is a type error). Adding the term $(h(x_i) - h_{S_i}(x_i))^2$ to J_{S_i} leaves its minimizer unchanged: the term is nonnegative and is zero there. Thus $h_{S_i} = h_{S_i'}$.

(b) Write $C = (K + \lambda I)^{-1}$ and $H = KC = CK$. The representer calculation, or first-order normal equations in the span of kernel sections, gives fitted values $Hy^{(i)}$ for the replaced-label vector. Symmetry of H gives

$$h_{S_i}(x_i) = e_i^\top H y^{(i)} = (y^{(i)})^\top CKe_i.$$

Let $t_i = h_{S_i}(x_i)$ and $\widehat{y}_i = h_S(x_i)$. Since $y^{(i)} = y + (t_i - y_i)e_i$,

$$t_i = \widehat{y}_i + H_{ii}(t_i - y_i), \qquad t_i - y_i = \frac{\widehat{y}_i - y_i}{1 - H_{ii}}.$$