Source task and conventions

Construct the empirical estimate and prove the stated high-probability bound.

Conventional solution and proof

Let $B_i = \mathbf{1}\{f(X_i) = +1\}$ for i.i.d. $X_i \sim D$ and set $\widehat{p} = m^{-1}\sum_i B_i$. Then $E\widehat{p} = p_+$. For completeness, if $B \sim \text{Bernoulli}(p)$, define $\psi(t) = \log Ee^{t(B-p)}$. Exponential tilting makes $\psi''(t)$ a Bernoulli variance, so $0 \le \psi''(t) \le 1/4$. Since $\psi(0) = \psi'(0) = 0$, integration twice gives $\psi(t) \le t^2/8$ for every real t. Independence and Markov's inequality imply

$$P(\widehat{p} - p \ge \varepsilon) \le \inf_{t>0} \exp(-tm\varepsilon + mt^2/8) = e^{-2m\varepsilon^2},$$

using $t = 4\varepsilon$. Apply the same argument to $-B_i$ and the union bound. Thus $P(|\widehat{p} - p| \ge \varepsilon) \le 2e^{-2m\varepsilon^2}$. For $0 < \delta < 1$, choosing $\varepsilon = \sqrt{\log(2/\delta)/(2m)}$ proves the requested confidence statement. Values $\delta \ge 1$ provide no substantive confidence claim.

Structural or analytic derivation

The moment-generating function packages all Bernoulli moments into one convex scalar function. Its curvature bound supplies a uniform tail certificate, while direct binomial summation provides an exact finite-m probability when p is known.

Comparative cost and preferred method