For $X \in [0,1]$, the tilted distribution still lies in $[0,1]$ and has variance at most $1/4$: $\operatorname{Var} X \le E(X-1/2)^2 \le 1/4$. The log-mgf curvature proof used in the label-bias solution therefore gives Hoeffding's bound $\min\{1, 2e^{-2m\varepsilon^2}\}$. Before clipping at one, Chebyshev is tighter exactly when

$$\sigma^2 < 2m\varepsilon^2 e^{-2m\varepsilon^2}.$$

After clipping, this strict comparison additionally requires the Chebyshev expression to be below one; otherwise the two displayed guarantees may simply tie at one.

(b) For Bernoulli variables $\sigma^2 = p(1-p) = 1/4 - (p-1/2)^2 \le 1/4$. At $\varepsilon = .05$, plot

$$C_m = \min(1, 100/m), \qquad H_m = \min(1, 2e^{-m/200}).$$

The enclosed graph uses integer $m = 1, \ldots, 3000$ and a logarithmic vertical scale. It shows a finite moderate-m range in which the variance bound is smaller, followed by exponential superiority for sufficiently large m. With dependence retained, the general Chebyshev bound must instead use the actual covariance sum $m^{-2}\sum_{i,j}\operatorname{Cov}(X_i, X_j)$; the common-variance assumption alone only bounds it by σ^2.

Structural or analytic derivation

The comparison is a comparison of certified tail envelopes at the same tolerance. Independence is a mathematical input, and the counterexample proves that it cannot be introduced implicitly.

Comparative cost and preferred method

Both bound evaluations are $O(1)$ for a specified m. Their useful difference is sample complexity: the worst-Bernoulli Chebyshev guarantee requires $m \ge 1/(4\delta\varepsilon^2)$, while Hoeffding requires $m \ge \log(2/\delta)/(2\varepsilon^2)$. Neither is uniformly the smaller bound for all known variances and sample sizes.