Source task and conventions

Prove the ridge property and determine the corresponding Euclidean-norm claims for lasso and successive PLS steps.

Conventional solution and proof

For $X = UDV^\top$ and $c = U^\top y$, ridge coefficients are $\beta_\lambda = \sum_j [d_j c_j/(d_j^2 + \lambda)]v_j$. Therefore

$$\|\beta_\lambda\|_2^2 = \sum \frac{d_j^2 c_j^2}{(d_j^2 + \lambda)^2},$$