## Source task and conventions

Derive the source basis change, explain its identifiability, and prove the fitted-value identity.

## Conventional solution and proof

Let $h(x) \in \mathbb{R}^M$ be the stated feature vector and fix the measure underlying the source's orthonormal eigenfunctions. Define $T = \int h(x)h(x)^\top \, d\mu(x)$ and diagonalize $T = VD_\gamma V^\top$. For $\gamma_j > 0$, put $\phi_j(x) = v_j^\top h(x)/\sqrt{\gamma_j}$. Then $\int \phi_j \phi_l \, d\mu = \delta_{jl}$ and

$$h(x) = VD_\gamma^{1/2}\phi(x), \qquad K(x,y) = \sum_j \gamma_j \phi_j(x)\phi_j(y),$$

on the nonnull feature span under the Mercer representation. Null directions are omitted; equality there is understood with the regularity/point-evaluation assumptions of the source. The kernel integral operator determines the $\gamma_j$ and $\phi_j$. Relative to a specified feature map, $v_j = \gamma_j^{-1/2} \int h(x)\phi_j(x) \, d\mu(x)$. The kernel alone does not determine the orientation $V$ of an unspecified feature map: replacing $h$ by an orthogonal rotation leaves $K$ unchanged. Nor do $N$ sample kernel values determine a global Mercer spectrum without a measure.

For coefficients $\beta$, let $c = D_\gamma^{1/2} V^\top \beta$. Then $\beta^\top h(x) = c^\top \phi(x)$ and the least attainable coefficient penalty is $\|\beta\|^2 = \sum_{\gamma_j > 0} c_j^2/\gamma_j$, with null coefficients set to zero. Substitution proves equivalence of the source criteria (5.53) and (5.63).