Source task and conventions

Identify the normalized limit and its one-component PLS interpretation.

Conventional solution and proof

Let $G = X^\top X$ and $c = X^\top y \neq 0$. Since $\beta_\lambda = (G + \lambda I)^{-1} c$, spectral diagonalization gives $\lambda \beta_\lambda \to c$ as $\lambda \to \infty$. Continuity of normalization away from zero therefore yields

$$\frac{\beta_\lambda}{\|\beta_\lambda\|} \longrightarrow \frac{c}{\|c\|}.$$

The first PLS score is $t = Xc$, and least squares on that single score has coefficient multiplier $a = t^\top y/(t^\top t) = c^\top c/(c^\top G c) > 0$. Thus its feature coefficient is $\beta_{\mathrm{PLS},1} = ac$, with the same normalized direction. Because $c \in \operatorname{ran} X^\top$, $c^\top G c > 0$ when $c \neq 0$. If $c = 0$, both coefficient vectors vanish and normalization is undefined; no directional limit is asserted.

Structural or analytic derivation

The resolvent expansion $(G + \lambda I)^{-1} = \lambda^{-1}[I - \lambda^{-1} G + O(\lambda^{-2})]$ identifies the first Krylov vector c as the leading term. The spectral proof supplies the limit without requiring a formal series outside its convergence range.

Comparative cost and preferred method