## Source task and conventions

Complete parts (a)–(c) for full row-rank $X = UDV^\top = RV^\top$.

## Conventional solution and proof

(a) Full row rank implies $R \in \mathbb{R}^{N\times N}$ is invertible. Every vector

$$\beta = VR^{-1}y + \eta, \qquad \eta \in \ker X,$$

has $X\beta = y$, and every zero-residual solution has this form by subtracting the displayed representative. Since $p > N$, the nullspace dimension is $p - N > 0$, giving infinitely many solutions. (b) Decompose $\beta = V\theta + \eta$. The ridge objective is $\|y - R\theta\|^2 + \lambda\|\theta\|^2 + \lambda\|\eta\|^2$. For $\lambda > 0$, $\eta = 0$ and differentiation gives $\theta = (R^\top R + \lambda I)^{-1}R^\top y$, the stated expression. (c) At zero penalty choose $\beta_0 = VR^{-1}y = VD^{-1}U^\top y$. It interpolates because $X\beta_0 = UDD^{-1}U^\top y = y$. Every other interpolant is $\beta_0 + \eta$ with $\eta \perp \beta_0$, so $\|\beta_0 + \eta\|^2 = \|\beta_0\|^2 + \|\eta\|^2$. This proves the claimed unique minimum-norm property.

## Structural or analytic derivation

Interpolation of the observed response and uniqueness of the feature coefficients are separate claims. The pseudoinverse selects a canonical coefficient vector by orthogonal projection; it does not establish predictive validity away from the observed row space.

## Comparative cost and preferred method