## Source task and conventions

Derive the complete local-polynomial cross-validated residual expression.

## Conventional solution and proof

At target $x_i$, fix the bandwidth, degree and basis $b_{ij} = (1, x_j - x_i, \ldots, (x_j - x_i)^d)^\top$, with nonnegative weights $w_{ij}$. Put $M_i = \sum_j w_{ij} b_{ij} b_{ij}^\top$ and $a_i = \sum_j w_{ij} b_{ij} y_j$. Assume the full and deleted local design matrices are invertible. The fitted value is $\widehat{f}_i = e_1^\top M_i^{-1} a_i$. Since $b_{ii} = e_1$, its self-weight is $S_{ii} = w_{ii} e_1^\top M_i^{-1} e_1$. Deletion gives $M_i^{(-i)} = M_i - w_{ii} e_1 e_1^\top$ and $a_i^{(-i)} = a_i - w_{ii} e_1 y_i$. The rank-one inverse formula follows by multiplying

$$(M - uu^\top)^{-1} = M^{-1} + \frac{M^{-1} uu^\top M^{-1}}{1 - u^\top M^{-1} u}$$

by $M - uu^\top$. Substituting $u = \sqrt{w_{ii}} e_1$ and simplifying gives

$$\boxed{y_i - \widehat{f}_i^{(-i)} = \frac{y_i - \widehat{f}_i}{1 - S_{ii}}, \qquad \mathrm{CVRSS} = \sum_i \left( \frac{y_i - \widehat{f}_i}{1 - S_{ii}} \right)^2 .}$$

Divide by $N$ for the mean error convention. The denominator is positive under the stated positive-definite deleted-design assumption. If $w_{ii} = 0$, the observation never enters its own fit and the formula reduces correctly to the ordinary residual.

## Structural or analytic derivation