eigenvalues lie in $[0, 1]$, including unpenalized components. Thus $0 \le S_{ii} < 1$ whenever the nondegenerate deletion identity applies, and

$$|y_i - \widehat{f}^{(-i)}(x_i)| = |y_i - \widehat{f}(x_i)|/(1 - S_{ii}) \ge |y_i - \widehat{f}(x_i)|.$$

At $S_{ii} = 1$, the fraction is undefined and the conclusion needs a separate interpolation/deletion analysis.

(c) Let the deleted linear prediction be $\widehat{f}^{(-i)}(x_i) = \sum_{j \ne i} a_{ij} y_j$, with both rules independent of response values. For $S_{ii} \ne 1$, the identity holds for every y if and only if

$$\boxed{a_{ij} = S_{ij}/(1 - S_{ii}) \quad (j \ne i).}$$

Necessity follows by comparing coefficients of each y_j; sufficiency follows by substitution. Equivalently the rule reproduces the deleted prediction after replacing the omitted response by that prediction. Linearity alone is insufficient: the full and deleted smoother families must have this relationship. Response-dependent parameter tuning generally destroys it.

Structural or analytic derivation

The rank-one normal-equation argument and the self-consistency argument are independent routes to the same deletion formula. The latter also covers spline penalties with an unpenalized nullspace when the fit remains unique.

Comparative cost and preferred method