Source task and conventions

Determine an admissible sampling rate, the sampled sequence, and the aliases when the angular sampling frequency is $10\Omega_0$.

Conventional solution and proof

Write $\Omega_s = 2\pi f_s$ and $T = 2\pi/\Omega_s$. The largest angular frequency is $11\Omega_0$, so an alias-free uniform sampling design for an arbitrary signal in this band requires

$$\boxed{\Omega_s > 22\Omega_0, \qquad f_s > 11\Omega_0/\pi.}$$

Strict inequality avoids the endpoint ambiguity at Nyquist. The sampled sequence is

$$x[n] = 3\sin(7\Omega_0 Tn) + 5\sin(5\Omega_0 Tn) + 7\sin(11\Omega_0 Tn).$$

At $\Omega_s = 10\Omega_0$, the three digital angular frequencies are $7\pi/5, \pi, 11\pi/5$. For every integer n, $\sin(\pi n) = 0$, $\sin(7\pi n/5) = -\sin(3\pi n/5)$ and $\sin(11\pi n/5) = \sin(\pi n/5)$. Thus

$$\boxed{x[n] = -3\sin(3\pi n/5) + 7\sin(\pi n/5).}$$

The second component is lost and the first and third are aliased. If the symbol "sampling frequency" denotes cycles per second, the value $10\Omega_0$ must first be converted; angular and cyclic units cannot be equated.

Structural or analytic derivation