The sampling map sends an analog exponential $e^{i\Omega t}$ to the sequence $(e^{i\Omega T})^n$. Two frequencies are indistinguishable exactly when their difference is an integer multiple of $2\pi/T$: equality at $n = 1$ proves necessity and integer powers prove sufficiency. Reducing the three frequencies modulo this lattice proves every alias above without drawing overlapping spectra. For a band-limited function whose spectrum is supported in $[-B, B]$, sampling with $2\pi/T > 2B$ places its translated spectral copies on disjoint intervals, which is the frequency-domain injectivity condition behind the recommended rate.

Comparative cost and preferred method

Both a careful spectral-folding baseline and the exponential reduction use three frequency reductions. For m specified tones the work is $O(m)$, matching the need to inspect all frequencies. The analytic representation supplies an exact alias identity; no runtime advantage over a competent conventional calculation is asserted.