Source task and conventions

Find the periods of $2\sin(\omega_0 n)$, $3\sin(3\omega_0 n)$ and $4\cos(4\omega_0 n)$ and their sum.

Conventional solution and proof

A nonzero sampled exponential has period N precisely when $\omega N \in 2\pi\mathbb{Z}$. Suppose $\omega_0/(2\pi) = p/q$ in lowest terms, with $q \geq 1$. The individual exponential periods are q, $q/\gcd(q,3)$ and $q/\gcd(q,4)$. A sampled sine with reduced denominator one or two is identically zero and is assigned fundamental period one; all other sine periods equal their exponential periods. The cosine has period $q/\gcd(q,4)$, including the constant case. The sum has the least common multiple of the nonzero-component periods. Here the unequal sine coefficients cannot cancel an aliased sine pair, and a real cosine coefficient cannot cancel an imaginary sine coefficient in the exponential expansion. Consequently its fundamental period is q for $q > 2$, and one for $q = 1, 2$. If $\omega_0/(2\pi)$ is irrational, all three components and their sum are aperiodic. Thus numerical periods cannot be given without ω_0.

Structural or analytic derivation

Expand each sine and cosine into the finitely many exponentials λ^n. Distinct exponentials are linearly independent: evaluating a linear relation at $n = 0, \ldots, r-1$ gives a Vandermonde matrix with determinant $\prod_{j<k}(\lambda_k - \lambda_j) \neq 0$. Periodicity of the sum therefore requires $\lambda^N = 1$ for each nonzero combined coefficient. This establishes the least-common-multiple statement, including aliases and degenerate zero sequences.

Comparative cost and preferred method