Source task and conventions

Compute $E_N = \sum_{n=0}^{N-1} \sin^2(\omega_0 n)$.

Conventional solution and proof

Using $\sin^2 u = (1 - \cos 2u)/2$,

$$E_N = \frac{N}{2} - \frac{1}{2}\Re \sum_{n=0}^{N-1} e^{2i\omega_0 n}.$$

If $\sin \omega_0 \neq 0$, multiplying the geometric sum by $1 - e^{2i\omega_0}$ and simplifying yields

$$\boxed{E_N = \frac{N}{2} - \frac{\sin(N\omega_0)\cos((N-1)\omega_0)}{2\sin\omega_0}.}$$

If $\omega_0 \in \pi\mathbb{Z}$, every sample vanishes and $E_N = 0$. The empty sequence $N = 0$ also has energy zero. For an integral number of nondegenerate half-periods, $\sin(N\omega_0) = 0$ and $E_N = N/2$.