Structural or analytic derivation

The identity is coefficient orthogonality on the unit circle. Writing the finite sum as $e^{i(N-1)\omega_0}\sin(N\omega_0)/\sin\omega_0$ explains both the energy and its exceptional denominator. Near a zero of $\sin\omega_0$, a scaled sinc evaluation or a series expansion avoids cancellation; the limiting formula alone is not a stable floating-point algorithm.

Comparative cost and preferred method

Direct summation uses N sine evaluations and N squares. The closed expression uses a fixed number of elementary-function evaluations for parameterized input. When the samples themselves are the input, reading them still costs $\Omega(N)$; no sublinear claim applies to arbitrary data.