Source task and conventions

For $x[-1], \ldots, x[4] = (1, 2, 1, 0, 1, 3)$, evaluate the DC value, the integral of the DTFT, and its squared-magnitude integral.

Conventional solution and proof

At zero frequency, $X(1) = \sum_n x[n] = 8$. For every integer m, $\int_{-\pi}^{\pi} e^{im\omega}\, d\omega$ is 2π if $m = 0$ and zero otherwise, as direct integration shows. Hence

$$\int_{-\pi}^{\pi} X(e^{i\omega})\, d\omega = 2\pi x[0] = 4\pi.$$

Expanding $|X|^2$ as a finite double sum and integrating kills all terms with unequal indices, giving

$$\boxed{\int_{-\pi}^{\pi} |X(e^{i\omega})|^2\, d\omega = 2\pi(1 + 4 + 1 + 0 + 1 + 9) = 32\pi.}$$

This is a complete finite proof of Parseval for this sequence.

Structural or analytic derivation

The three requested outputs are respectively evaluation at one, the constant Laurent coefficient, and the coefficient inner product. The index origin is essential: the constant coefficient is two, not the first listed value one.

Comparative cost and preferred method