Conventional solution and proof

By the inverse DFT and the substitution $r = (k-4)_8$,

$$v[n] = \frac{1}{8}\sum_r W[r]e^{2\pi i(r+4)n/8} = e^{i\pi n}w[n] = (-1)^n w[n].$$

Thus

$$\boxed{v = (2, 0, 1, -3, 2, -1, 4, -1).}$$

Every index, including the wrap at $k = 0$, is covered by the periodic exponential identity.

Structural or analytic derivation

The shift of frequency coordinates is the character $n \mapsto (-1)^n$ acting on the original samples. It requires sign changes only; no root calculation or transform evaluation remains.

Comparative cost and preferred method

Exactly the odd-index entries change sign. For a general even length N, a half-spectrum shift takes at most $N/2$ negations, versus two transforms for a direct frequency-domain reconstruction. This achieves linear output complexity.