Source task and conventions

Evaluate $X[0]$, $X[4]$, $\sum_k X[k]$ and $\sum_k |X[k]|^2$ for the given w, treating X as its DFT.

Conventional solution and proof

Directly from the definition,

$$X[0] = 2 + 0 + 1 + 3 + 2 + 1 + 4 + 1 = 14,$$
$$X[4] = 2 - 0 + 1 - 3 + 2 - 1 + 4 - 1 = 4.$$

Summing the DFT over k and using roots-of-unity orthogonality gives $\sum_{k=0}^{7} X[k] = 8w[0] = 16$. Expanding the squared magnitude and using the same identity gives

$$\boxed{\sum_{k=0}^{7} |X[k]|^2 = 8 \sum_{n} |w[n]|^2 = 8(36) = 288.}$$

The source changes the name W to X and prints a mismatched summation index; the intended eight frequency values are specified here.

Structural or analytic derivation

The outputs are evaluation at the trivial and alternating characters, coefficient extraction at zero, and the unitary-norm identity. All are invariants of the DFT matrix and can be proved without computing its other six entries.

Comparative cost and preferred method