Source task and conventions

Compute both DFTs, their product, its inverse and the circulant-matrix convolution for $x = (1, 2, 0, 1)$ and $h = (2, 1, 0, 1)$.

Conventional solution and proof

With $e^{-2\pi i/4} = -i$,

$$X = (4, 1 - i, -2, 1 + i), \quad H = (4, 2, 0, 2), \quad Y = (16, 2 - 2i, 0, 2 + 2i).$$

The inverse formula $y[n] = \frac{1}{4} \sum_{k=0}^{3} Y[k] e^{2\pi i k n/4}$ gives

$$\boxed{y = (5, 5, 3, 3).}$$

The circular convolution matrix is

$$C_h = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{pmatrix}, \qquad C_h x = (5, 5, 3, 3)^T.$$

Each row implements $\sum_m h[(n - m)_4] x[m]$, verifying part (iv) independently.

Structural or analytic derivation