Consequently the complete answer is

$$\boxed{G = (1+2i,\ (\tfrac{1}{2}-i)e^{-4\pi i/5},\ 3e^{-8\pi i/5},\ (\tfrac{1}{2}+i)e^{-12\pi i/5},\ (1-2i)e^{-16\pi i/5}).}$$

No reality assumption on h is needed, and the supplied spectrum does not satisfy the usual real-sequence symmetry at DC.

Structural or analytic derivation

Multiplication by a monomial z^{-2} in the circular time algebra becomes multiplication by a diagonal unit-modulus matrix in frequency coordinates. Thus magnitudes are unchanged and phase shifts linearly by $-4\pi k/5$.

Comparative cost and preferred method

Five scalar phase factors replace an inverse DFT followed by a shift and another DFT. For arbitrary N, this is $O(N)$ instead of $O(N \log N)$ transform work, and the output-size lower bound is $\Omega(N)$.