Source task and conventions

Derive the length-four FFT flow and apply it to $(1, 0, 1, 0)$.

Conventional solution and proof

Split the defining sum into even and odd time indices. Define

$$a = x_0 + x_2, \quad b = x_0 - x_2, \quad c = x_1 + x_3, \quad d = x_1 - x_3.$$

The four outputs are

$$\boxed{X_0 = a + c, \quad X_2 = a - c, \quad X_1 = b - id, \quad X_3 = b + id.}$$

Substituting the input gives $(a, b, c, d) = (2, 0, 0, 0)$ and $X = (2, 0, 2, 0)$. The diagram records the two stages and the quarter-turn multiplication. There are eight complex additions/subtractions, no general complex multiplications, and one multiplication by $-i$, implemented by exchanging real/imaginary components and changing a sign. In the required two-stage radix-two butterfly architecture, each of four butterflies needs its sum and difference, so eight additions are necessary within that architecture. No lower bound for arbitrary unrestricted linear circuits is asserted.

Structural or analytic derivation

The algebra is the factorization of the four-point Fourier matrix into two block-diagonal two-point Fourier transforms, a diagonal twiddle and a permutation. The identity $(-i)^{2k} = (-1)^k$ proves the even/odd split directly and handles all four output indices.