Source task and conventions

Give a block construction that uses an existing forward DFT routine to compute an inverse DFT.

Conventional solution and proof

For the negative-exponent forward transform F, complex conjugation of its defining sum yields

$$\boxed{F^{-1}Y = \frac{1}{N}\,\overline{F(\overline{Y})}.}$$

Indeed the right-hand component is $N^{-1}\sum_k Y_k e^{2\pi ikn/N}$, exactly the inverse definition. The block chain is input $Y \to$ conjugate $\to F \to$ conjugate $\to$ scale by $1/N$. Alternatively $F^{-1}Y = N^{-1}R(FY)$, where R is circular reversal; its proof is the same exponent substitution. The scale is essential.

Structural or analytic derivation

The identity $F^2 = NR$ follows from roots-of-unity orthogonality. Since $R^2 = I$, it provides an algebraic inverse and a verification test independent of any stored inverse-transform implementation.

Comparative cost and preferred method

The method requires one forward transform plus $O(N)$ conjugations/scales or a reversal. It has the same asymptotic cost as a dedicated inverse FFT and reduces the number of independently maintained transform kernels from two to one. That maintenance reduction is not a measured runtime speedup.