Source task and conventions

Transform the three exponential sequences in Problem 2.6.1, with $u[n] = 1$ for $n \geq 0$.

Conventional solution and proof

For (i), with $|\beta| < 1$,

$$W(e^{i\omega}) = \sum_{n=2}^{\infty} (\beta e^{-i\omega})^n = \frac{\beta^2 e^{-2i\omega}}{1 - \beta e^{-i\omega}}.$$

For (ii), with integer $L \geq 0$,

$$X(e^{i\omega}) = 1 + \sum_{n=1}^{L} \beta^n (e^{-in\omega} + e^{in\omega}) = 1 + \frac{r_-(1 - r_-^L)}{1 - r_-} + \frac{r_+(1 - r_+^L)}{1 - r_+}, \quad r_\pm = \beta e^{\pm i\omega}.$$

When a denominator vanishes, its finite-sum term is L; the polynomial expression is valid for every finite β. For (iii), put $m = -n \geq 4$ and use $\beta > 1$:

$$Y(e^{i\omega}) = \sum_{m=4}^{\infty} (\beta^{-1} e^{i\omega})^m = \frac{\beta^{-4} e^{4i\omega}}{1 - \beta^{-1} e^{i\omega}}.$$

The geometric sum follows by multiplying its partial sum by $1 - r$ and passing to the limit only when $|r| < 1$.

Structural or analytic derivation