All three calculations are evaluations of Laurent polynomials or Laurent series on $|z| = 1$. The right-sided sequence needs $|z| > |\beta|$, and the left-sided sequence needs $|z| < \beta$. Their Fourier transforms exist because the unit circle lies in the respective convergence region. The finite symmetric sequence combines two analytic geometric polynomials, so apparent poles at $r_{\pm} = 1$ are removable.

Comparative cost and preferred method

The infinite sums have fixed-size exact expressions with explicit convergence conditions. The finite direct sum needs $O(L)$ terms per frequency; geometric evaluation takes $O(\log L)$ multiplications for integer powers, or fixed transcendental calls in a floating-point model. The usual geometric-series method is equally efficient. The source's $\beta < 1$ in (i) is qualified to $|\beta| < 1$.