Source task and conventions

Express the DTFT of $x[-n]$ in terms of the DTFT of a real sequence $x[n]$.

Conventional solution and proof

Changing variable $m = -n$ gives $\sum_n x[-n]e^{-i\omega n} = \sum_m x[m]e^{i\omega m} = X(e^{-i\omega})$. Since $x[m]$ is real, conjugating the defining sum gives $X(e^{-i\omega}) = \overline{X(e^{i\omega})}$. Thus the answer is $\boxed{X(e^{-i\omega}) = X^*(e^{i\omega})}$. The argument holds pointwise for absolutely summable sequences and in the L^2 sense for square-summable sequences.

Structural or analytic derivation

Reflection on the unit circle sends z to $\bar{z} = z^{-1}$. Real coefficient symmetry identifies this reflection with complex conjugation. The proof contains the relevant convergence convention and does not assume the transform itself is real.

Comparative cost and preferred method

A stored spectrum needs only conjugation, $O(N)$ entries, instead of a new transform costing $O(N \log N)$ by FFT on a finite grid. If only one frequency is requested and its value is known, one conjugation suffices. The standard Fourier symmetry is already the preferable method.