## Source task and conventions

Find all three Z transforms and their exact regions of convergence.

## Conventional solution and proof

For (i), summing from $n = 2$ gives

$$X_1(z) = \frac{(1/2)^2 z^{-2}}{1 - (1/2)z^{-1}}, \quad |z| > 1/2.$$

For a left-sided exponential $a^n u[-n-3]$, let $m = -n \geq 3$. Then

$$\sum_{n=-\infty}^{-3} a^n z^{-n} = \sum_{m=3}^{\infty} (z/a)^m = \frac{(z/a)^3}{1 - z/a}, \quad |z| < |a|.$$

Hence (ii) is $-\frac{(z/(2/5))^3}{1-z/(2/5)}$ with $|z| < 2/5$, and (iii) is $\frac{(z/(-2/5))^3}{1-z/(-2/5)}$ with $|z| < 2/5$. The origin is included for these strictly left-supported polynomial-at-zero expressions.

## Structural or analytic derivation

The three rational expressions are accompanied by distinct Laurent expansion domains. The sign of $a$ moves the pole but does not change the radius of convergence. Finite shifts change polynomial factors and the behaviour at zero or infinity, while the nonzero finite boundary radius remains $|a|$.

## Comparative cost and preferred method