Source task and conventions

Derive the Z transform of $r^n \cos(\omega_0 n)u[n]$ and specialize to $2^n \cos(\pi n/2)u[n]$.

Conventional solution and proof

Writing the cosine as two exponentials and summing gives

$$Y(z) = \frac{1}{2}\left(\frac{1}{1 - re^{i\omega_0} z^{-1}} + \frac{1}{1 - re^{-i\omega_0} z^{-1}}\right) = \boxed{\frac{1 - r\cos\omega_0 z^{-1}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}}, \quad |z| > |r|.$$

For $r = 2$ and $\omega_0 = \pi/2$,

$$\boxed{W(z) = \frac{1}{1 + 4z^{-2}}, \quad |z| > 2.}$$

The sequence is $1, 0, -4, 0, 16, 0, -64, \ldots$, confirming its expansion. For $r = 0$, the sequence is just $\delta[n]$ under the convention $0^0 = 1$ and the apparent region restriction is removable.

Structural or analytic derivation

The poles are $re^{\pm i\omega_0}$. This is the elementary spectral decomposition of a second-order recurrence; conjugate poles ensure a real sequence. Combining the two fractions before evaluating can reduce arithmetic but does not authorize use of the formula on the unit circle when $|r| \geq 1$.

Comparative cost and preferred method