Two geometric series give a fixed-size rational result. Computing a long prefix by the second-order recurrence is $O(N)$; evaluating the rational transform at one admissible point has fixed arithmetic cost. The standard exponential decomposition is already efficient.