Source task and conventions

Decide whether $2^n u[-n-1] + 3^n u[n+1]$ has a bilateral Z transform.

Conventional solution and proof

The first summand is supported on $n \le -1$ and requires $|z| < 2$. The second begins at $n = -1$ and has an infinite positive tail requiring $|z| > 3$. Their intersection is empty. More directly, on the positive tail the terms $(3/z)^n$ fail to tend to zero unless $|z| > 3$; on the negative tail the terms $(z/2)^m$ fail unless $|z| < 2$. These conditions cannot both hold. The overlap at $n = -1$ changes one finite coefficient and cannot cancel either tail. Therefore no nonempty annulus of bilateral convergence exists.

Structural or analytic derivation

Algebraically adding the two separately continued rational formulas gives a meromorphic expression, but it is not the sum of the original bilateral series on any common domain. Analytic continuation does not create a convergence region for a different specified Laurent series.