There are two nonzero simple poles. Their residues require fixed arithmetic for any chosen index, with powers computed in $O(\log |n|)$ multiplications. A full prefix uses an $O(N)$ recurrence. Partial fractions are a strong conventional control and yield the same outputs; contour quadrature would add unnecessary numerical error here.