Source task and conventions

Invert $H(z) = (1 - z^{-1}/2)/[(1 - z^{-1}/3)(1 - 2z^{-1})^2]$ and examine the printed condition $|z| > 1/3$.

Conventional solution and proof

The exact decomposition is

$$H(z) = -\frac{1/50}{1 - z^{-1}/3} + \frac{3/25}{1 - 2z^{-1}} + \frac{9/10}{(1 - 2z^{-1})^2}.$$

It has a noncancelled pole at $z = 2$, so the entire set $|z| > 1/3$ cannot be a region of analyticity or convergence. Two valid regions satisfying its lower-radius intention give different inverses. For $|z| > 2$,

$$h[n] = \left[-\frac{1}{50} 3^{-n} + \left(\frac{3}{25} + \frac{9}{10}(n + 1) \right) 2^n \right] u[n].$$

For $1/3 < |z| < 2$,

$$h[n] = -\frac{1}{50} 3^{-n} u[n] + \left[-\frac{3}{25} + \frac{9}{10}(-n - 1) \right] 2^n u[-n - 1].$$

In the second expression the repeated-pole contribution is zero at $n = -1$, as required. There is no unique inverse under the printed region alone.

Structural or analytic derivation

For a right-sided double pole, differentiating the geometric series yields $(1 - r)^{-2} = \sum_{n \geq 0} (n + 1) r^n$. For a left-sided expansion, $(1 - 2/z)^{-2} = (z/2)^2 (1 - z/2)^{-2}$, whose coefficient at $n \leq -2$ is $(-n - 1)2^n$. These two expansions prove both inverses and explain why pole multiplicity produces a linear factor in the index.