Source task and conventions

Find the first five causal impulse samples for $H(z) = z/(3z^2 - 4z + 1)$.

Conventional solution and proof

Put $q = z^{-1}$ and write $H(q) = q/(3-4q+q^2) = \sum_{n\geq 0} h_n q^n$. Equating coefficients gives $3h_0 = 0$, $3h_1 - 4h_0 = 1$ and $3h_n - 4h_{n-1} + h_{n-2} = 0$ for $n \geq 2$. Thus

$$\boxed{h_0 = 0,\ h_1 = \tfrac{1}{3},\ h_2 = \tfrac{4}{9},\ h_3 = \tfrac{13}{27},\ h_4 = \tfrac{40}{81}.}$$

This coefficient recurrence is long division with exact fractions. It also proves uniqueness of the causal expansion.

Structural or analytic derivation

Factoring the denominator gives $H = \frac{1}{2}z/(z-1) - \frac{1}{2}z/(z-1/3)$, so $h_n = \frac{1}{2}(1 - 3^{-n})u[n]$. Substitution reproduces all five coefficients. The causal region is $|z| > 1$; the impulse response tends to $1/2$, so the system is not BIBO stable.

Comparative cost and preferred method

For five outputs, recurrence/long division is the preferred $O(N)$ method. A closed form helps remote indices but does not reduce the output lower bound for a complete prefix. The pole at one gives a stability conclusion without simulating a long response.