Source task and conventions

Obtain the transfer function for $h[n] = \sum_{k=0}^{3} \alpha_k \delta[n-k]$.

Conventional solution and proof

The defining Z transform contains only one nonzero contribution from each delta:

$$\boxed{H(z) = \alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \alpha_3 z^{-3}.}$$

The input-output relation is $y[n] = \sum_{k=0}^{3} \alpha_k x[n-k]$. The nonzero finite plane is a convergence domain; zero is excluded when a positive-delay coefficient is nonzero, and any formal poles created by multiplying numerator and denominator are not extra physical modes.