Source task and conventions

Derive the impulse response and transfer function of the causal M-point moving average.

Conventional solution and proof

Apply the system to $x[n] = \delta[n]$. This gives $h[n] = 1/M$ for $0 \le n < M$ and zero otherwise. Therefore

$$\boxed{H(z) = \frac{1}{M} \sum_{k=0}^{M-1} z^{-k} = \frac{1 - z^{-M}}{M(1 - z^{-1})}.}$$

The rational form has a removable value at $z = 1$, namely one; the true filter is FIR. On the unit circle,

$$H(e^{i\omega}) = \frac{1}{M} e^{-i(M-1)\omega/2} \frac{\sin(M\omega/2)}{\sin(\omega/2)},$$

with its continuous limiting values at multiples of 2π. Multiplication of the finite sum by $1 - z^{-1}$ proves both forms.

Structural or analytic derivation

The zeros are the nontrivial Mth roots of unity. The formal integrator pole in the rational expression cancels exactly, explaining why the rolling-sum realization can be BIBO stable as an input-output filter while its internal accumulator needs finite-precision care.

Comparative cost and preferred method