Conventional solution and proof

Conjugate symmetry gives $X^*(e^{i\omega}) = X(e^{-i\omega})$, whose inverse is $x[-n]$. Linearity then gives

$$\boxed{\mathcal{F}^{-1}(\Re X)[n] = \tfrac{1}{2}(x[n] + x[-n]), \quad \mathcal{F}^{-1}(\Im X)[n] = \frac{x[n] - x[-n]}{2i}.}$$

The second inverse is generally imaginary; the inverse of $i\Im X$ is the real odd part. This follows directly from $\Re X = (X + X^*)/2$ and $\Im X = (X - X^*)/(2i)$. For a finite DFT use circular reversal $[-n]_N$.

Structural or analytic derivation

The two projections $P_e = (I + R)/2$ and $P_o = (I - R)/2$, where $Rx[n] = x[-n]$, satisfy $P_e^2 = P_e$, $P_o^2 = P_o$, $P_e P_o = 0$. They decompose the time sequence into even and odd components. The factor $1/i$ is required by the chosen transform convention.

Comparative cost and preferred method

Pairwise sums and differences cost $O(N)$ and use no inverse FFT. The saving against two fresh transforms is asymptotically $O(N \log N)$ to $O(N)$ for finite arrays, with the same outputs. The source mixes DTFT and DFT terminology; both conventions are supplied explicitly.