Source task and conventions

Prove $X(\omega) = X^*(-\omega)$ and the even/odd symmetries of its components.

Conventional solution and proof

For real $x[n]$, conjugating $\sum_n x[n]e^{i\omega n}$ gives $\sum_n x[n]e^{-i\omega n} = X(\omega)$. Therefore $X(-\omega) = \overline{X(\omega)}$. Taking real parts proves $\Re X(-\omega) = \Re X(\omega)$; taking imaginary parts proves $\Im X(-\omega) = -\Im X(\omega)$. This supplies all three assertions directly, including zeros and endpoints. The same absolute-summability or L^2 qualification as in the preceding identities applies.

Structural or analytic derivation

In a Laurent representation, reality of the coefficients is precisely invariance under $X(z) \mapsto \overline{X(\bar{z})}$. On the unit circle the reflection changes the frequency sign. No symmetry of the original time sequence is required.

Comparative cost and preferred method

Only one half of a symmetric frequency grid needs independent evaluation. Completing the other half costs conjugations; the number of independent values is reduced by approximately two, with DC and Nyquist values real. This is a standard transform property, not a separate novel acceleration.