Source task and conventions

Test the claimed linear phase of $h = (1, -2, 3, 2, -1)$.

Conventional solution and proof

The displayed coefficients give

$$H(e^{i\omega}) = e^{-2i\omega}\{3 + i[2\sin 2\omega - 4\sin\omega]\}.$$

Set $b(\omega) = 2\sin 2\omega - 4\sin\omega$. Since the real part of the bracket is three, it never vanishes and its continuous argument is $\arctan(b/3)$. Thus

$$\phi'(\omega) = -2 + \frac{3b'(\omega)}{9 + b(\omega)^2}.$$

At zero, $b = 0, b' = 0$, so $\phi'(0) = -2$. At π, $b = 0, b' = 8$, so $\phi'(\pi) = 2/3$. These different derivatives prove that the phase is not affine. Consequently the requested assertion is false for the printed filter.

Structural or analytic derivation

For a real FIR polynomial with nonzero endpoints, exact generalized linear phase requires symmetric or antisymmetric coefficients. This sequence has neither: $h_0 = -h_4$, $h_1 = -h_3$, but the middle coefficient is nonzero, which violates antisymmetry; the endpoints violate symmetry. The direct derivative calculation supplies an independent counterexample without relying solely on that characterization.