Source task and conventions

Prove or correct all four identities in Problem 2.6.7.

Conventional solution and proof

Let $X(\omega) = \sum_n x[n]e^{-i\omega n}$. Reindexing gives $\mathcal{F}\{x[-n]\} = X(-\omega)$. Reindexing and conjugating gives

$$\mathcal{F}\{x^*[-n]\} = \sum_m x^*[m]e^{i\omega m} = X^*(\omega).$$

The source's second identity omits the conjugation on the result. For $x[n] = i\delta[n]$, its two sides would be $-i$ and i, an explicit contradiction. Also $\mathcal{F}\{x^*[n]\} = X^*(-\omega)$. Expanding real and imaginary parts now proves

$$\mathcal{F}\{\Re x\} = \tfrac{1}{2}[X(\omega) + X^*(-\omega)], \quad \mathcal{F}\{i\Im x\} = \tfrac{1}{2}[X(\omega) - X^*(-\omega)].$$

All manipulations are justified by absolute summability, or by density and continuity for ℓ^2 transforms.

Structural or analytic derivation

Conjugation C, reversal R and their product are commuting involutions. Their projections recover the real/imaginary and even/odd subspaces. Tracking both operations prevents the missing conjugation in part (ii).

Comparative cost and preferred method