Source task and conventions

Determine absolute summability and energy of $x[n] = A\alpha^n u[n]$.

Conventional solution and proof

Assume A is finite. If $A = 0$, both sums vanish. Otherwise

$$\sum_{n\in\mathbb{Z}} |x[n]| = |A| \sum_{n=0}^{\infty} |\alpha|^n, \qquad E = \sum_n |x[n]|^2 = |A|^2 \sum_{n=0}^{\infty} |\alpha|^{2n}.$$

For $|\alpha| < 1$, multiplying each geometric series by $1 - |\alpha|$ or $1 - |\alpha|^2$ gives

$$\boxed{\|x\|_1 = \frac{|A|}{1-|\alpha|}, \qquad E = \frac{|A|^2}{1-|\alpha|^2}.}$$

For $|\alpha| \geq 1$, the nonzero summands fail to tend to zero, so neither sum converges. The printed assumption $\alpha < 1$ alone allows $\alpha = -2$ and is insufficient.

Structural or analytic derivation

The transfer generating function $X(z) = A/(1 - \alpha z^{-1})$ converges for $|z| > |\alpha|$. Its pole is inside the unit disk exactly in the summable case. Parseval gives the same energy from the unit-circle integral; the geometric proof above already establishes the needed convergence and is shorter for one pole.

Comparative cost and preferred method