A truncation at N requires N terms and leaves exact energy tail $|A|^2|\alpha|^{2N}/(1-|\alpha|^2)$. The closed form uses fixed arithmetic and gives the entire infinite sum. The conventional geometric-series solution is optimal for this symbolic input; complex analysis adds a pole interpretation.

Source qualification

The convergence condition is stated in modulus; this is an explicit qualification of the source's inequality.